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MATHEMATICAL MODEL OF A LTV EVAPORATOR

M. BECCARI, A. C. DI PINTO and M. SANTORI
Consiglio Nazionale delle Ricerche Istituto di Ricerca Sulle Acque, Rome, Italy

and

L. BIASI, A. PROSPERETTI and A. TOZZI
Applicazioni e Ricerche Scientifiche S.p.A., Milan, Italy

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INTRODUCTION

A new mathematical description of a Long Tube Vertical (LTV) evaporator is formulated by introducing Sleicher's turbulence model (Davis 1965) for the region close to the tube wall. As a consequence, the momentum and energy equations can analytically be integrated with respect to the radial coordinate. Differently from the previous formulations of Dukler (1960) and Kroll–McCutchan (1968) who both used Deissler's turbulence model, the problem is here reduced to the integration of a system of first order ordinary differential equations, describing the variation of the various quantities along the tube.

DESCRIPTION OF THE MODEL

The liquid and vapor distribution shown in figure 1 has been assumed in the present paper. The following assumptions are made:

(a) no liquid entrainment in the vapor;
(b) no waves on the film surface;
(c) the shear stress distribution in the film is linear;
(d) the film thickness Δ is negligible with respect to the tube radius R;

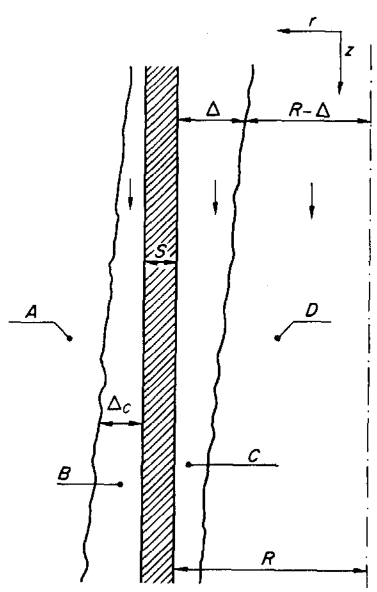

Figure 1. Liquid and vapor distribution. A: Condensing vapor; B: Condensed film; C: Evaporating brine film; D: Vapor.

(e) evaporation takes place at the film surface in equilibrium with the local temperature T and pressure p;
(f) liquid motion is turbulent and the velocity u is in equilibrium with the local value of the quantities on which it depends;
(g) the momentum and energy turbulent transport in the film is described by Sleicher's model in the region close to the tube wall and Von Kàrmàn's model in the remaining part of the film. The eddy diffusivity ϵ is therefore given by the following expressions:

$$\epsilon(y^+) = \begin{cases} A\nu y^{+2} & y^+ \le 26 \\ \chi^2 \nu \left(\frac{\partial u^+}{\partial y^+}\right)^3 \Big/ \left(\frac{\partial^2 u^+}{\partial y^{+2}}\right)^2 & y^+ > 26 \end{cases} \qquad [1]$$

where $y^+ = [\sqrt{(\tau_w/\rho)}/\nu]y$ is a dimensionless distance from the tube wall; y is the distance from the tube wall; τ_w is the wall shear stress: ρ is the liquid density; ν is the kinematic liquid viscosity; A is a dimensionless constant equal to 0.091; χ is Von Kàrmàn's universal constant equal to 0.4.

Assumptions (a) and (b) have been introduced in order to simplify the computation of liquid film thickness and flow rate. Their influence will be partially taken into account in the computation of the friction pressure drop.

Assumption (f) is of basic importance for the present formulation since it allows calculation of the local liquid film heat transfer coefficient h and flow rate Γ as a function of the local value of the interface shear stress.

Assumption (g) allows calculation of an analytic relationship between film flow rate, local heat transfer coefficient and film thickness.

In the framework of the assumptions made, the momentum and energy equations can be integrated analytically with respect to the radial coordinate, yielding the following expressions for the film flow rate and the heat transfer coefficient:

$$\frac{\Gamma}{2\pi R\nu} = \begin{cases} \frac{1}{A}\left(\eta - \frac{\sigma^3}{A\eta^2}\right)\tan^{-1}(A\eta) + \frac{\sigma^3}{A^2} + \\ -\frac{1+\sigma^3}{2A^2}\ln(1 + A^2\eta^2) & \eta \le 26; \\ \frac{2c_1\eta}{\chi\sigma^3}[c_2(c_2 - c_1/2)\ln c_1 + c_1(c_1/3 - 3/4c_2)] + \\ -(c_3 - u^+_{26})(\eta - 26) - c_4 + \frac{\Gamma_{26}}{2\pi R\nu} & \eta > 26; \end{cases} \qquad [2]$$

$$\frac{h}{(gk^3/\nu^2)^{1/3}} = \begin{cases} \frac{1}{(\sigma^3/\eta)^{1/3}} \frac{A\sqrt{(Pr)}}{\tan^{-1}(A\sqrt{(Pr)}\eta)} & \eta \le 26; \\ 1\Big/\left\{\frac{(gk^3/\nu^2)^{1/3}}{h_{26}} + \frac{(\sigma^3/\eta)^{1/3}}{Pr\chi c_2}\ln\left[\frac{c_1}{c_3 - \sqrt{(1-26\sigma^3/\eta)}} \times \right.\right. \\ \left.\left.\times \frac{c_2 - c_3 + \sqrt{(1-26\sigma^3/\eta)}}{c_2 - c_1}\right]\right\} & \eta > 26; \end{cases} \qquad [3]$$

where: $\eta = [\sqrt{(\tau_w/\rho)}/\nu]\Delta$ is a dimensionless film thickness; $\sigma^3 = 1_L - \tau_i/\tau_w$; τ_i is the interface shear stress; $u^+ = [u/\sqrt{(\tau_w/\rho)}]$ is a dimensionless local film velocity; $c_2 = \sqrt{(1-26\sigma^3/\eta)} + (1+26A^2/2\chi)(\sigma^3/\eta/1 - 26\sigma^3/\eta)$; $c_1 = c_2 - \sqrt{(1-\sigma^3)}$; $c_3 = 1/\chi[c_2\ln(c_2 - \sqrt{(1-26\sigma^3/\eta)}) + \sqrt{(1-26\sigma^3/\eta)}]$; $c_4 = [2(c_3 - \sqrt{(1-26\sigma^3/\eta)})/\chi\sigma^3/\eta]\{c_2[c_2 - (c_3 - \sqrt{(1-26\sigma^3/\eta)})/2]$ ln

$(c_3 - \sqrt{(1-26\sigma^3/\eta)}) + (c_3 - \sqrt{(1-26\sigma^3/\eta)})[(c_3 - \sqrt{(1-26\sigma^3/\eta)})/3 - 3c_2/4]\}$.

Expressions [2] and [3] are valid for both evaporating and condensing films and reproduce the relationship between film flow rate, heat transfer coefficient, interfacial shear stress and film thickness reported in graphical form by Dukler (1960). The total heat transfer coefficient is computed by taking into account the two films and tube wall heat resistence.

The liquid film flow rate variation along the tube axis is given by the two equations:

$$\frac{d\Gamma_e}{dz} = -\frac{2\pi R}{H_c} h_t (T_c - T_e), \qquad [4]$$

$$\frac{d\Gamma_c}{dz} = -\frac{H_e}{H_c}\frac{d\Gamma_e}{dz}, \qquad [5]$$

where Γ_e is the evaporating film flow rate; Γ_c is the condensing film flow rate; H is the evaporation enthalpy; z is the axial coordinate; h_t is the total heat transfer coefficient; T_c is the condensing liquid temperature; T_e is the evaporating liquid temperature.

Temperatures T_e and T_c are intended to be computed at the liquid–vapor interface.

For the assumption (e) the two temperatures T_e and T_c depend only upon the pressure through Clausius–Clapeyron equation. Generally, T_c can be assumed as a constant and known among the plant characteristics.

Inside the tube, the pressure is a function of the liquid film flow rate through the relationship:

$$\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_{fr} + \rho_v g - \frac{1}{A_v}\left[\frac{d}{dz}\left(\frac{\Gamma_v}{\rho_v A_v}\right)^2 - u_{R-\Delta}\frac{d\Gamma_v}{dz}\right] \qquad [6]$$

where $(dp/dz)_{fr}$ is the friction pressure drop; ρ_v is the vapor density; $A_v = \pi(R-\Delta)^2$ is the vapor flow area; $u_{R-\Delta}$ is the interface velocity.

Expression [6] has been obtained through integration of the momentum equation in the radial direction with the assumption that the square mean velocity is equal to the mean square one.

The last term on the RHS represents the acceleration contribution to the total pressure gradient and can have a magnitude equal or also greater than the friction term. On the other hand, it can be remarked that the liquid film flow rate and heat transfer coefficient only depend upon the shear stress at the liquid–vapor interface and therefore uniquely upon the friction pressure drop. This latter can be computed with the aid of one among the several correlations known in the literature.

RESULTS AND INFLUENCE OF PROCESS VARIABLES

The ordinary differential equations [4] to [6] together with [2] and [3] and the equilibrium one, fully describe the operations in every element of a LTV plant. Once the conditions at the tube inlet or outlet are given, the numerical integration of this system of equations provides the local value of the main characteristic quantities such as pressure, temperature, liquid film flow rate and heat transfer coefficient.

The model predictions have been compared with the experimental data of Wrightsville Beach (Badger & Associated, Inc. 1959) and Freeport (Stearns Roger Co. 1964). Since the conditions for these plants were only known at the outlet, the equations were integrated starting from the outlet and going backwards up to the point where the condensing film was exhausted. The evaporation length computed in this way was checked to be always less (about 20%) than the effective tube length.

A more interesting quantitative comparison with the experimental data of I.R.S.A. plant in Bari (Beccari *et al.* 1973) is shown in table 1. For these data, conditions were known at the inlet and the evaporating film was always entering at its boiling point. The first column data were obtained using distributors of the cone type whereas for the second column data a vortex

Table 1

			U(W/m^2°C)		
Re_0	T_0(°C)	ΔT_0(°C)	Cone	Vortex	Model
2555	100.5	4.0	—	4743	4377
4989	100.2	4.5	3692	5292	4516
7540	100.2	4.5	3676	5283	4646
2555	100.6	5.8	—	4665	4417
4989	100.4	6.0	3599	4975	4546
7540	100.5	5.9	3716	5109	4672
2550	111.5	3.9	—	4961	4437
4966	111.2	4.2	3634	5504	4562
7526	111.2	4.4	3878	5544	4696

arrangement was used. The better agreement of the model with this latter data seems to be due to a more uniform distribution of the evaporating film. For all comparisons, the friction pressure drop was computed using Lockart–Martinelli's correlation (1949).

In order to put in evidence the dependence of the overall heat transfer coefficient U upon the main process variables, a parametric study was performed. The region that was investigated is characterized by the values shown in table 2. Computation of variable main effects and their first order interactions showed that U has a monotonous dependence upon inlet Reynolds number

Table 2

$2500 \leq Re_0 \leq 7500$
$50°C \leq T_0 \leq 125°C$
$3°C \leq \Delta T_0 \leq 9°C$
$2\,m \leq L \leq 6\,m$
o.d. = 5.08 cm
i.d. = 4.88 cm

Re_0, temperature T_0 and temperature drop ΔT_0. It does not present a monotonous trend with respect to the tube length L. The most important interactions resulted those between length and temperature drop, length and film Reynolds number and length and temperature. The results of this parametric study thus provided the following expression for the overall heat transfer coefficient:

$$U(\mathrm{W/m^2 °C}) = 201\frac{Re_0 - 2500}{2500} + 267\frac{T_0 - 50}{25} - 178\frac{\Delta T_0 - 3}{3} + 97\left(\frac{\Delta T_0 - 3}{3}\right)^2 - 97\frac{L-2}{2} + 69\left(\frac{L-2}{2}\right)^2 + 43\frac{(Re_0 - 2500)(L-2)}{5000} - 96\frac{(T_0 - 50)(L-2)}{50} + 141\frac{(\Delta T_0 - 3)(L-2)}{6} + 4114. \quad [7]$$

Expression [7] reproduces the model predictions with a mean square deviation of 4%, allows an easy computation of the overall heat transfer coefficient and can be fruitfully used in the computation of an optimum plant design.

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